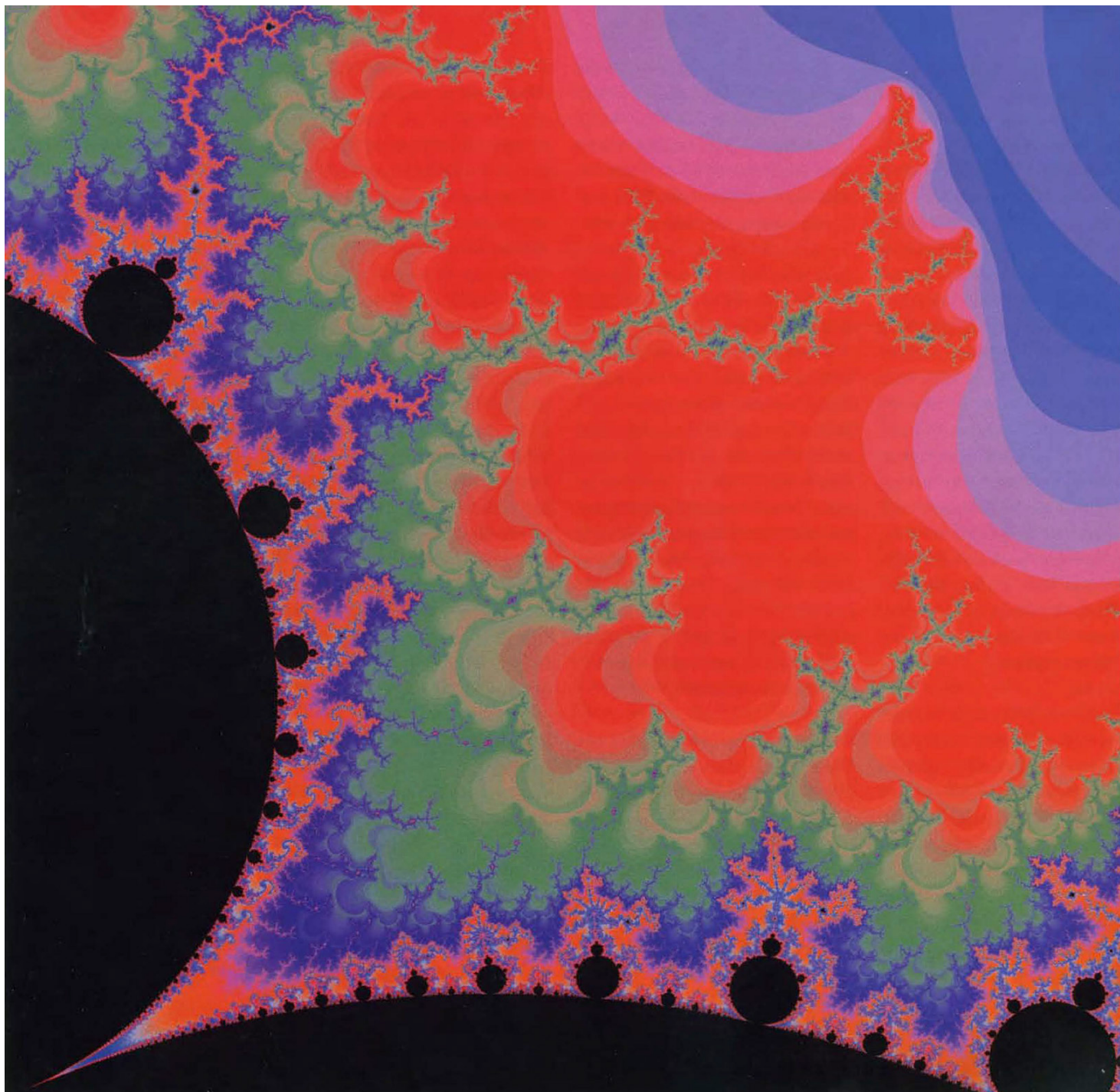




The Fractal Geometry of Experience

only a cloth around his middle, in his hand a switch. With a gentle rhythmic motion he swung the switch, alternately tapping the sides of the buffalo's hump. This scene sent an electric shock through my system. To the finest detail, it was one that had not changed in four thousand years. This seemed not simply a vision *of* antiquity, but a vision *from* antiquity. It was like a physical projection of something buried deep in my unconscious. I had much the same reaction when I first encountered the Mandelbrot Set.

In the 1970s at IBM's Watson Research Center in northern Westchester County, Benoit Mandelbrot developed a new geometry he called "fractal geometry." It is not a geometry of idealized forms, of squares, cones, and spheres. It is a geometry that describes many of the basic complex structures of nature: the branching of a tree or of our circulatory system; the cracking of city pavement as it undergoes the stress of weather, light, and wear; the billowing of cumulus clouds over the plains; the jaggedness of the coast of Maine; the erosion of the Grand Canyon. It is the geometry of the forms of

growth and decay, dealing with structures that progress to finer and finer layers of detail and yet maintain a similar form. When viewing the Grand Canyon on any scale - from a satellite or a high-flying plane, a hot air balloon or a second-story window at the Canyon Lodge, from our eye level or from the vantage point of a child on hands and knees examining a fissure in the baked earth at the canyon's rim - we see the same kinds of forms, the same ruggedness, the same degree of complexity. This quality, "scaling," is a central feature of both nature and of fractal geometry. It is the patterns of ever-finer branching, of eddies within eddies, of organic fragmentation, patterns that fill our natural world, that are the stuff of fractal geometry.

While living in Paris in the 1950s, Benoit Mandelbrot became interested in the patterns of the frequency of word use. He had immigrated to France from his native Poland in the 1930s, fleeing the Holocaust with his family. An uncle who was a famous mathematician lived and taught in Paris and encouraged him in pursuing mathematics. That uncle's interest

in the theoretical was, for Mandelbrot, balanced by his father's love of the practical. It was the combination of the theoretical and the practical that formed the basis for Mandelbrot's investigations. He wanted to apply mathematical analysis to the world he saw around him. And so, he found himself, almost by chance, drawn to the study of word frequencies. There was, as he would put it, a smell about it that intrigued him. He recognized this smell years later when he was studying economics.

His first interest in economics was income distribution. He then studied commodity prices, finding himself attracted to phenomena that were not described by bell curves (in which the greatest frequency of occurrences clusters around the average with diminishing frequency as one moves away from the average). He began to realize that what drew him to these fields of study, what was behind the smell that attracted him, was a common pattern and this was the pattern of scaling. To demonstrate how scaling applies to commodity prices, take, for example, the market price of cotton. A graph of

Stephen Shore

Several years ago, on a sere North African January afternoon, I was traveling to the pyramids at Sakkara north of Cairo. Climbing out of the fertile Nile Valley, heading toward the desert, I passed through a transitional zone, a mile-wide band of tall, stately palms rising from the desert floor. Through these palms was a broad beaten path. Slowly advancing up the path were a herd of water buffalo, with one buffalo in the lead. Astride him was a young boy, wearing



Wynn Bullock, *Erosion*, 1956

hourly price fluctuations would look the same as a graph of daily price fluctuations, which, in turn, would look the same as a graph of weekly price fluctuations, which, in turn, would look the same as a graph of monthly price fluctuations. This structural self-similarity on different scales is scaling. Mandelbrot knew that this was significant. What follows from this understanding? he asked himself. He explored other disciplines including fluid dynamics and hydrology and kept running into self-similarity - scaling.

Mandelbrot had always thought visually. He tells a story of once having a geometry professor who used a textbook without illustrations. The professor said that images lied; that a circle was best described as $x^2 + y^2 = 1$. For Mandelbrot, a circle was described by that equation and by: O. They are different aspects of the same reality. Underlying his analytic perceptions were mental images, which he felt helped unlock his intuition. He would have a visual understanding of an aspect of the world and then, unlike the artist for whom visual understandings are communicated visually, he would

work to devise a formula that would delineate the phenomenon, turning mental images into numbers. With the advent of computer graphics in the late 1960s, the numbers were turned back into images. The computer graphics would, in turn, refresh his intuition, to use Mandelbrot's words. Working with graphics paved the way to fractal geometry. This is same interplay of intuition, analysis, and computer graphics led years later, in the late 1970s, to the discovery of the Mandelbrot Set.

Mandelbrot was working with the formulae of two early twentieth-century French mathematicians, Pierre Fatou and Gaston Julia. Their work involved iterated formulae, in which the result of an operation is fed back into the formula, whose operation in turn is repeated, and so on. Their formulae are plotted on the complex plane - a mathematical plane whose two axes are the real numbers and the imaginary numbers.¹ The work of Fatou and Julia couldn't be taken further until the development of the computer, because the computer enables almost endless iteration. It was while playing with what are called Julia Sets and searching for a basic

formula of which the Julia Sets are only special instances that Mandelbrot discovered the set that bears his name.

The Mandelbrot Set is a visual representation of a boundary within a specific mathematical mapping. It is a mathematical object of beauty, power, and, literally, infinite complexity. The set begins with an island like shape. At one end the two sides curve in toward the central axis. On the opposite end there is a spire projecting out along the central axis. Floating on the spire is a much smaller version of the original island. Shooting like solar flares from the perimeter of the big island are whorls and sea horse tails, paisleys and lightning bolts. No matter how much any segment of the perimeter is magnified - a hundred times, a thousand times, a million times - similar forms continue to appear, similar but not identical. Every now and then a shape like the original island reappears and on its edge the process begins all over again.

What is especially compelling for me about the Mandelbrot Set is a quality of recognition, the sense that I've seen it before. Mandelbrot

reports that many people have had this same experience. It is far less surprising that fractals in general and the phenomenon of scaling should spark recognition, they so fill the world in which our species has evolved and in which we, as individuals, have grown up. Very few shapes in nature look Euclidean. It would take a complicated Euclidean formula to describe a complex shape such as a lunar mountain. Yet this same shape could be described by a simple fractal formula. Fractal geometry brings the discipline of geometry back to the meaning of its Greek root, *geometria*, to measure the world. Fractal geometry did not spring from abstract thought, but from observation of the real world. This very fact made it unappealing at first to mathematicians, but engaging to a lay public. Mandelbrot believes that people love mathematical structure even if they don't realize they do. This is another way of saying that there is a basic analytic component to our nature. We need to find order in the world, in some rudimentary way, to be able to function in it. On the simplest level this may entail the conceptualization of a fact: my dog learning the word

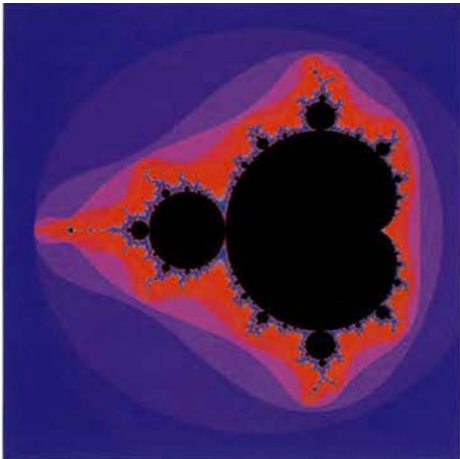


Image 1:

The Mandelbrot Set, shown in its entirety, is contained in this image, which is roughly two inches on each side. Bound between -2.25 and 0.75 on the x axis, it extends only 1.5 units above and below on the y axis.

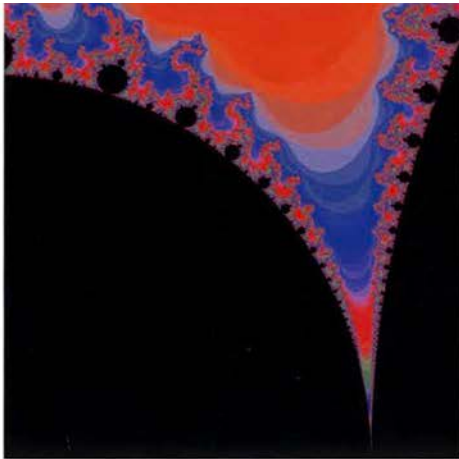


Image 2:

As the image is magnified by a factor of ten, we see detail emerging from the needle-shaped forms near the center of the set. The seemingly circular shapes on each side are repeated in an infinitely decreasing pattern.

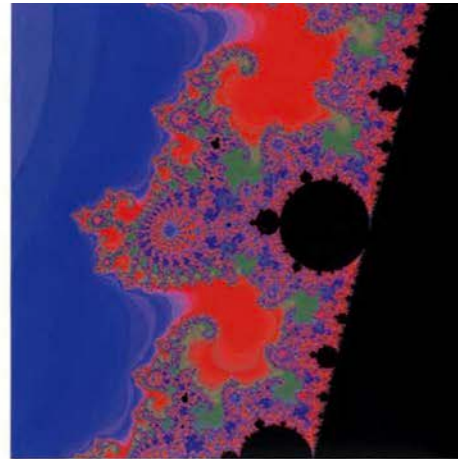


Image 3:

Another zoom by ten allows closer inspection of a circle. Rather than becoming smoother, more detail is revealed as the set is examined with greater magnification. If the original image were shown at this scale, it would be nearly seventeen feet on each side.



Image 4:

Here, a radial pattern is displayed. Spiraling in infinite detail towards the center, this image exhibits a complexity only hinted at in the original portrait of the Mandelbrot Set, whose sides would now be more than one-half the length of a football field.

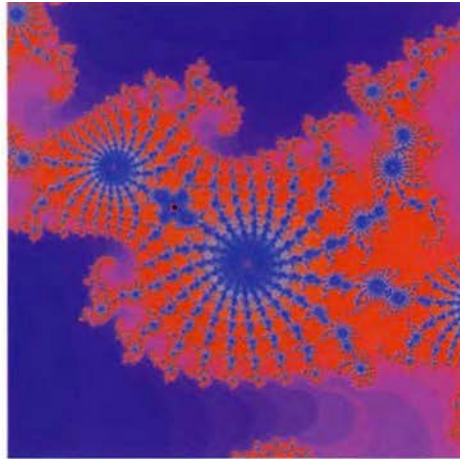


Image 5:

After another zoom by a factor of ten, the symmetry of the previous image is reflected in two smaller portions of the set. Although these are miniature versions of the pattern, there is no breakdown of detail as we continue to look more closely at the set.

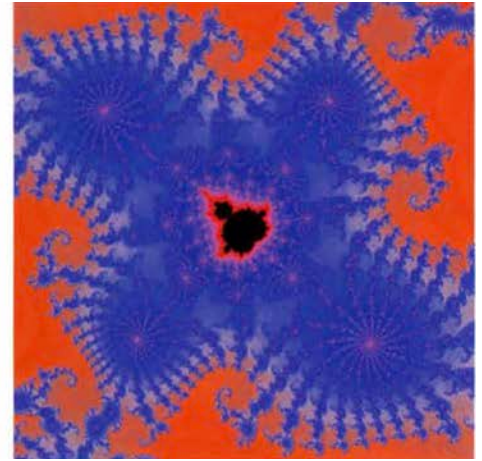
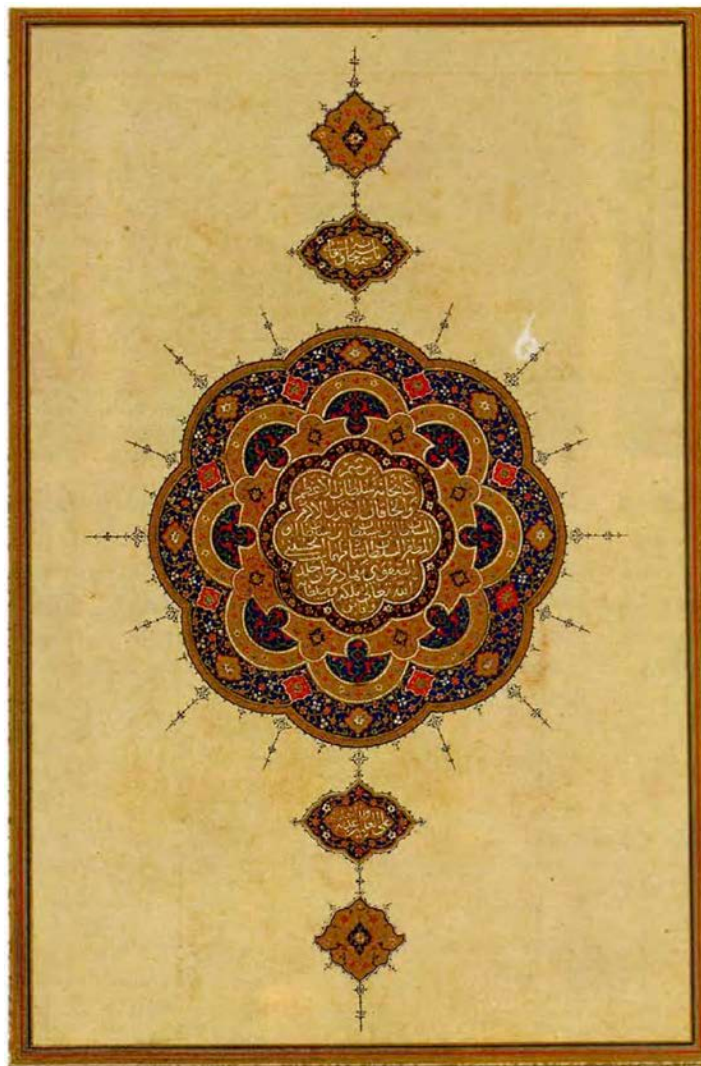


Image 6:

The original image has now become more than three miles on a side. If displayed at the scale of the first image, this would be microscopic. However, the level of detail has not diminished, and, in fact, a replica of the original Mandelbrot Set has appeared that contains all the detail of its ancestor. The appearance of this miniature illustrates the self-similarity of the Mandelbrot Set, which will never cease to provide infinite variation at any magnification.

“road” (as in “Get out of the road!”) and recognizing each instance of a road he encounters. On a more profound level this ordering reflects our world view, be it simplistic and rigid or complex and fluid. And it is the analytic part of us that is stimulated by a visual representation of a geometrical structure that so well correlates with our experience of the world.

Phenomena so pervasive in the world as fractals and scaling must seat themselves at the core of the structure of our understanding. Artists through the ages have consciously or intuitively understood the visual aspect and power of these phenomena. From medieval illuminations and Persian miniatures to the application of paint on a Cézanne landscape, artists have depicted fractal forms. Artists were attracted to these forms not simply because they describe nature, but because they “mean something.” For the Chinese landscape painter, fractals described the effervescence of a life force in mountains or the poignancy of a lone pine. For the builders of the Gothic cathedrals, fractals were the language of the complexity in unity of God’s cre-



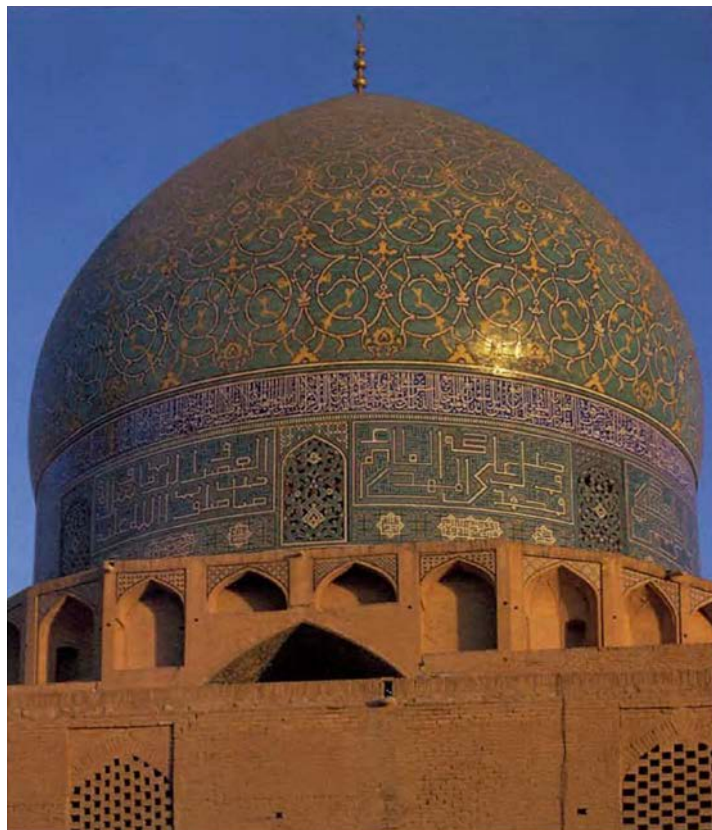
Dedicatory page of the Houghton Shah-Nemah (16th century)

ation. For Dürer, in “Knight, Death and the Devil,” the fractal cliffs and exposed roots looming over the scene alluded to time, death, and decay. But they are not simply symbols, they are analogs of deeper meaning and experience. They exist for what they are and at the same time resonate on psychological, emotional, and spiritual levels.

There are also some artistic creations that bear a similarity to the Mandelbrot Set in whole or in part. This is harder to explain than an artist’s attraction to fractals, since the Mandelbrot Set is a mathematical object and unlike fractals doesn’t exist in the world of our experience.² Perhaps the most obvious example is the paisley. Its complex curving and swirling forms, their density and repetition and scaling properties are all strikingly similar to the activity deep within the perimeter of the set.

Another example is the architectural dome topped by a spire. Instances are found in many cultures. In most cases the spire is interrupted by a small sphere or other shape much as the spire of the Mandelbrot Set is interrupted

by a small repetition of the island. The schematic architectural roots of this may be found in Stupa # 1 at Sanchi, India (3rd to 1st century B.C.). Consider St. Basil's in Moscow with its onion domes (16th century). Here the bulging sides and interrupted spires are clearly reminiscent of the set. To this is added the complex patterning on the domes' surfaces. Complex patterning on spired domes also appears throughout Islamic architecture. It is as though the prominences and whorls of the Mandelbrot Set arc collapsed into arabesques on the domes' surfaces. Freed from the physical constraints of architecture, Islamic calligraphers and artists illuminated manuscripts and Korans with spired medallions and rosettes complete with complex edges. Take the dedicatory page of the *Houghton Shah-Nemah* (16th century) reproduced on the opposite page. Here is the delineated central axis, the spires interrupted by small rosettes, the prominences from the edges of both large and small rosettes, the richly intricate layering of pattern on different scales. Each small section presents a world dense with structure into which one can enter. The upper



Masjid-i-Shaykh, Isfahan

cartouche reads: "In His Name, the Most Praised and Most Exalted!" That these images and domes appear repeatedly in a sacred context should underline the centrality of their meaning to their cultures.

For the artist, fractals do not simply represent the look of nature, they are an expression, a manifestation, of a force deep within nature, and, by extension, within ourselves, from our circulatory systems and bronchial passages to

our nervous systems and brains. And to our minds. The image of the Mandelbrot Set was a source from which some artists and craftsmen of different cultures and periods drew in the creation of their sacred buildings and pictures. We are attracted to it and find it strangely familiar because it is an image that we hold in our unconscious minds.

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Notes:

1. An imaginary number is the square root of a negative real number, e.g., $\sqrt{-4}=2i$
2. It does bear a relationship to something called the bifurcation diagram. The bifurcation diagram illustrates the onset of chaos in a system and has applications in biology, electronics, optics, chemistry and other fields. The bifurcation diagram is a slice on the real plane through the Mandelbrot Set displays its phantasmagorical forms.